

Metamaterial Textures

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Figure 1: When an external force is applied, metamaterial textures undergo a controlled transformation. This door handle, for example, transforms (a) from flat (b) to rippled (c) to spiky, allowing the person behind the door to set a tactile message with three levels of *enter/busy/do not enter* messages for visually impaired or sighted users trying to enter.

ABSTRACT

We present metamaterial textures—3D printed surface geometries that can perform a controlled transition between two or more textures. *Metamaterial textures* are integrated into 3D printed objects and allow designing how the object interacts with the environment and the user’s tactile sense. Inspired by foldable paper sheets (“origami”) and surface wrinkling, our 3D printed metamaterial textures consist of a grid of cells that fold when compressed by an external global force. Unlike origami, however, metamaterial textures offer full control over the transformation, such as in between states and sequence of actuation. This allows for integrating multiple textures and makes them useful, e.g., for exploring parameters in the rapid prototyping of textures. Metamaterial textures are also robust enough to allow the resulting objects to be grasped, pushed, or stood on. This allows us to make objects, such as a shoe sole that transforms from flat to treaded, a textured door handle that provides tactile feedback to visually impaired users, and a configurable bicycle grip. We present an editor assists users in creating metamaterial textures interactively by arranging cells, applying forces, and previewing their deformation.

Author Keywords

Metamaterials; textures; fabrication; 3D printing; programmable matter.

ACM Classification Keywords

H.5.m. [Information interfaces and presentation] Misc.

INTRODUCTION

Digital fabrication machines such as 3D printers excel at producing arbitrary shapes, such as for decorative objects. Recently, researchers proposed going beyond designing merely the external shape, but to divide materials into a large number of cells and to design each cell’s structure to perform a specific deformation [27, 33]. Such structures are also known as *metamaterials*, which are “artificial structures with mechanical properties that are defined by their usually repetitive cell patterns, rather than the material they are made of” [37]. The ability to design each cell individually allows for literally thousands of degrees of freedom.

Researchers used the concept of metamaterials to design mechanical properties of materials, e.g., vary the stiffness across an object [54], make materials contract in two dimensions when compressed in one dimension [7, 56], damp an impact [55], or change the shape and surface of materials on a macroscopic [6, 35] or microscopic scale [11, 67]. More recently, researchers in HCI applied the concept of metamaterials to create 3D printed objects that implement mechanisms [15] and simple computation [16].

In this paper, we apply the main idea behind metamaterials, i.e., subdivision into a large number of cells and customization on a per-cell basis, to the *outsides* of 3D printed objects. The resulting *metamaterial textures* allow designers to shape how the object interacts with the environment and with the tactile sense of the user.

METAMATERIAL TEXTURES

Metamaterial textures are 3D printed surface geometries that can perform a controlled transition between two or more textures. Haptic properties, such as compliance [54], weight, and static texture [61, 19] can enhance 3D objects and are easy to fabricate. More complex fabrication machines, such as multi-material 3D printers, also allow for continuously controllable textures [11]. However, so far, they do not apply *objects* and are limited by one texture.

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In this work, we introduce metamaterials that undergo a controlled transformation when an external force is applied, resulting in *multiple* dynamic textures. Figure 1 shows an example. This door handle transforms from flat to rippled to spiky, allowing the person behind the door to set three levels of *enter/busy/do not enter* tactile messages for everyone trying to enter. For completeness, we integrated our textured handle with our metamaterial door latch [15].

As shown in Figure 2, the inside of the door handle consists of a grid of cells, which controls how the texture on the object’s surface will be formed.

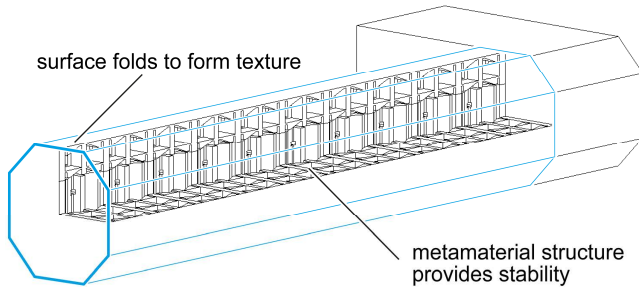


Figure 2: Metamaterial textures are made from cells that can fold upwards, creating a tactile bump. The metamaterial allows for this behavior while simultaneously providing stability.

Figure 3 illustrates the design of the underlying cells, which we call *fold cells*. Each cell implements a simple mechanism that transforms horizontal compression into vertical deformation, i.e., it *folds* upwards when compressed. The cell consists of two four-bars, which is a basic linkage the rigid members of which move in parallel. When the cell is compressed horizontally it causes the four-bars to shear and the cell to fold upwards, creating a tactile bump. Hence, chaining multiple of these cells allows *poping out a texture* on the surface of the object. These four-bars can be repeated inside the object until it is filled.

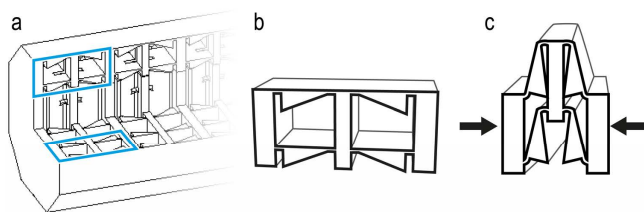


Figure 3: (a) Textured objects consist of many (b) unit cells, which (c) pop out of the object’s surface, when compressed.

Metamaterial textures are generally actuated to transition between textures by a global compression. To ease user interaction, we deploy them with a mechanism that allows producing the force required to deform the metamaterial texture. Figure 4 shows the mechanism we use to actuate the door handle texture in Figure 1. The mechanism runs strings through the door handle. As the user turns the knob, the strings are wound up and cause all fold cells to compress and to fold outward, forming the texture. During actuation metamaterial textures compress by a certain amount—30% in the case of the door handle example in Figure 1.

This means our approach is limited to objects for which length is not a critical property.

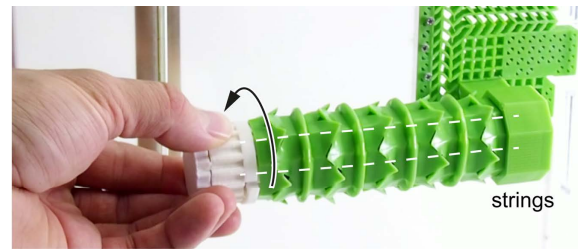


Figure 4: Users transition through the door handle’s embedded textures by turning the knob. That winds up the strings on the inside, which compresses all cells and forms the textures.

Application examples

Metamaterial textures are suitable for conveying tactile messages or providing tactile feedback, for rapid prototyping of textured objects, or for adapting objects on demand.

1. Conveying tactile messages. Figure 1 showed an example for this category as a door handle that utilizes its surface to inform users (visually impaired or sighted) about one’s availability for interruptions. We achieve three textures within the one object by (1) alternating the two textures row-wise and (2) defining the sequence in which they pop out based on the amount of compression, i.e., compressing the door handle halfway only activates the smoother ridges, and compressing it all the way adds the spiky texture to the previous bumpy texture.

2. Adapting the functionality of an object to the context of use. Figure 5 shows a shoe sole with a metamaterial texture. It can be transformed from a flat sole to a corrugated one for more traction on snow or mud. This example illustrates one benefit of these metamaterial structures: they add enough *stability* to the origami-like surface to hold the weight of an adult (here 55 kg). To achieve this, we designed our cell to fold tightly, which prevents the beams from buckling in order to maximize strength (Figure 3c).

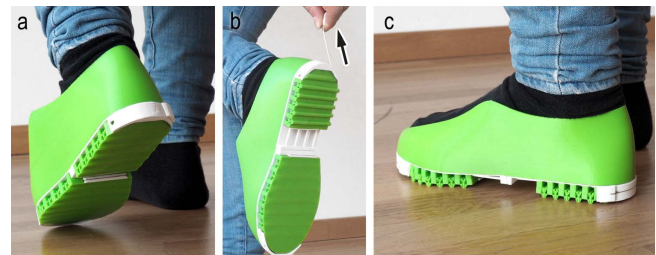


Figure 5: (a) This shoe sole is flat by default. (b) The user transforms it into a treaded sole it by pulling a string, e.g., when it starts snowing. (c) Note that the sole is functional and robust enough to walk on.

3. Exploring texture designs quickly. One of the qualities of metamaterial textures is that they can be continuously actuated to different levels (Figure 6a-c). This is useful when trying to explore how “strong” a texture should be during rapid prototyping. To test for the ergonomics of this bicycle grip, designers can prevent cells locally from folding. Fig-

ure 6d shows that sliding spacers into the material causes those cells to resist the compression. Designers can then again explore if the design feels right (Figure 6e).



Figure 6: (a) Designers fabricate one single bicycle grip that they (b) actuate continuously to different levels, (c) to feel the tactile qualities during rapid prototyping. (d) By inserting spacers after fabrication that (e) deactivates selected rows allows them to further investigate the grip’s ergonomics.

Metamaterial textures allow product designers to quickly iterate through multiple textures in one 3D print only, instead of fabricating many prototypes, which is slow. In fact, designers and researchers agree that tactile designs “need to be felt early and often” [52]. Similar approaches exist for quickly iterating over rigid 3D shapes [25], we extend this idea to texture prototyping.

CONTRIBUTIONS

Our main contribution is the concept of embedding *multiple* dynamic textures into one 3D printed object using metamaterials. Such textured objects allow for continuously transitioning between magnitudes of the texture. Furthermore, they allow users to define the sequence in which the cells fold, which enables transitioning between multiple integrated textures after fabrication.

Our textures are integrated in the object at printing time. We contribute parameterized metamaterial cells that function using a single material and that enable a range of textures by only varying the cell geometry.

To assist users and researchers in designing new textures using metamaterials, we contribute an interactive editor that features a fast preview of the texture transformation.

DESIGN SPACE OF METAMATERIAL TEXTURES

To summarize the capabilities of metamaterial textures, we characterize their design space, as illustrated in Figure 7. The resulting design space consists of six dimensions:

Single cell primitives: We identified three geometry classes that can be parameterized to create a range of shapes for the tactile bump on a single cell: simple straight ridges (which can create box-like cells or rounded cells), diagonal (zigzag, spiky, etc.) or diamond shaped.

Composition: Single cell primitives can be composed to form a texture by (1) *uniformly tiling* the same cells one next to the other. A more expressive composition can be achieved by (2) chaining cells of the same type *row-wise* (such as in our door handle example). Lastly, it is also possible to (3) compose the texture from a *cell-wise* arrangement, the only restriction being that the positions of the folding hinge must join continuously from one cell to the next in the row.

Spacing: We identified three possible variations for how to define the space between tactile bumps. The simplest configuration available to designers is to spread out their bumps at equally distant points by *uniformly* spacing cells. To *vary* the spacing between tactile bumps, designers can choose cell parameters which increase or decrease the spacing and chain these cells. A complete explanation of the cell parameters is given in section “Amplitude and frequency of a texture”. Lastly, the distance between bumps can be increased by inserting *spacers* into the material after it was fabricated, which prevents the selected cells from folding.

Transformation: Designers can define how the transformation between textures will be performed: the texture can fold in parallel, sequentially, or a combination of these. A parallel transformation implies that all cells fold at once (e.g., the bike grip example). A sequential transformation causes cells to fold subsequently as the force increases, as illustrated by the door handle in Figure 1. This is detailed in “Force-dependent textures”.

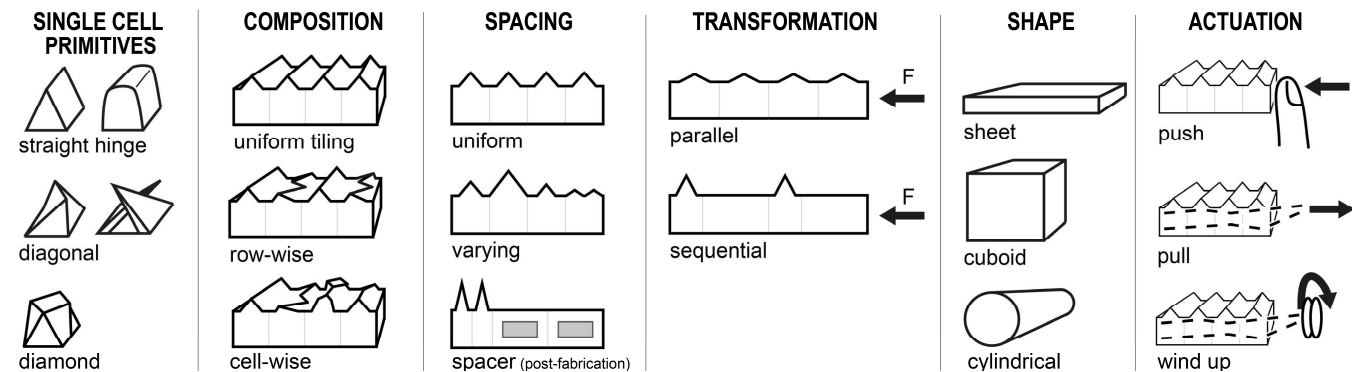


Figure 7: Our six-dimensional design space describing metamaterial textures.

Shape: We identified three shape classes that our metamaterial textures can cover. The simplest shape that metamaterial textures can be applied to is a planar sheet. It can also be applied to cylindrical shapes (e.g., bike grip) or on the outside of cuboid shapes, e.g., the door handle example.

Actuation: Since our textures are actuated by global compression, we see three actuation possibilities for our resulting textures. The simplest form of actuating metamaterial textures is by pushing them manually to compress. Alternatively, designers can run strings through the material that are either pulled or wound up. While in this paper we explore the idea of having no electronics in our materials and use human actuation for all three actuation classes, using motors or other automated actuators is certainly possible.

RELATED WORK

Our work builds on previous work in personal fabrication, shape changing interfaces, haptic research on textures and mechanical metamaterials.

Personal fabrication

Personal fabrication machines such as 3D printers and laser cutters allow users to fabricate personalized physical objects. To also enable novice users to work with these machines, researchers in HCI proposed systems that create shapes interactively [10, 25, 26, 42, 62, 63]. To add sensing to 3D printed objects, researchers introduced techniques that integrate electronics into these objects by, for example, adding pipes on the inside for light transmission [51], adding makers for optical tracking of internal movements [50], printing optics [63], or capacitive sensing [1, 17].

Recently, the haptic properties of 3D printed objects attracted the attention of researchers, who investigated tactile properties such as textures [59], weight, or compliance [61] for digitally fabricated *static* objects. Since in interaction and product design, the “feel” of an object is a crucial matter, researchers proposed devices that can fabricate objects from materials other than rigid plastic, e.g., by enabling printers to use felt [38], wool [14], or combining textiles with plastic [45].

Shape-changing interfaces

Objects that when actuated change their external shape are often denoted as “shape-changing interfaces” [43]. These are typically used to represent data. One canonical example is *shape displays*, which are used mostly for tangible data representation (e.g., *Inform* [8]) but also to assemble objects [53] or even to allow designers emulate material properties, such as viscosity [28]. To do so, these 2.5D displays actuate every pixel’s height (a pin which is moved by means of an actuator, typically a motorized fader). As the interest in handheld shape changing interfaces increases, researchers showed how to curl or bend objects by folding the structure and using global actuation such as pneumatics [32, 49, 65] or cables [29, 40, 48]. Further miniaturization can be achieved by actuating thin sheets with electricity [13, 47], by electro-magnetism [60], or by embedding

bacteria that changes the cell’s geometry when exposed to moisture [66].

We build on these works, but we focus on dynamic textures that are integrated into the 3D printed material itself. Furthermore, our metamaterial structures that support the textures are strong enough for users to grab or even stand on.

Dynamic textures in Haptics

In the field of haptics, the ultimate goal is emulation of the real world’s physical sensations: kinematic/proprioceptive and tactile. The latter is the human sense that perceives the texture of objects.

Haptic icons based on textures are able to represent information to users while they explore a textured surface [20]. Researchers in haptics have shown how users perceptually organize these icons according to shape, amplitude and frequency. MacLean et al. showed that perceptual interpolation methods exist between texture profiles for a force-feedback knob [21]. Clark et al. later generalized this to arbitrary amplitude profiles and validated it using vibrotactile sensations [4]. Klatzky et al. studied how the density of a texture’s bumps affects the perceived roughness of it [18]. The ability to dynamically create textures on demand is crucial to communicate more information to the user’s tactile sense. Haptic devices that emulate textures include VR/AR haptic gloves that vibrate the user’s fingertips [3] or tactile displays for the blind, such as braille arrays [39] or line printers [58]. These hardware approaches are fast and precise but bulky due to their mechanical actuators, which are unfeasible to integrate in objects printed with consumer grade personal fabrication devices.

Our approach is inspired by the concepts of dynamic texture for communication (e.g., door handle that informs users) but achieves this without resorting to electronics.

Mechanical metamaterials

Metamaterials are commonly understood as “artificial structures with mechanical properties that are defined by their usually repetitive cell patterns, rather than the material they are made of” [37].

Researchers in mechanical engineering have used this concept to alter mechanical properties of structures, e.g., alter the stiffness of objects [33, 35, 46], alter elasticity [12], or damp impacts [55]. Furthermore, metamaterials are used to embed “unusual” behavior in the material itself, e.g., structures that pull in the direction of compression rather than resisting it (negative stiffness) [5, 40] or objects that contract in two dimensions when exposed to compression on a single dimension (auxetic materials) [7, 23, 56].

Textured materials attracted attention for their ability to tune mechanical properties in a continuous and reversible manner. Popping up one pre-defined texture from pre-cut sheets (“kirigami”) changes the material’s stiffness [41] and shape [29]. For tuning micromechanical properties, such as drag reduction, researchers proposed embedding rigid parti-

cles within a soft material [11, 67], which cause the surface to wrinkle upon compression and form a micro-texture.

In computer graphics, researchers adopted the concept of metamaterials and changed how objects feel, e.g., by emulating different materials (such as leather or felt) from plastic [2], by varying the stiffness across an object [54], by animating objects [6, 36], or by designing foams that compress differently in different directions [22]. Recently, in HCI, interactive metamaterials were explored by appropriating metamaterials for integrating functionality into interactive objects without the need for electronics. For instance, Ion et al. integrated mechanical functions into materials without the need for moving parts [15], or integrated simple computing functions in the material itself [16], e.g., adding a combination lock to a metamaterial door latch.

Our approach is rooted in these works and extends them by embedding *multiple* controllable dynamic textures based on metamaterials into single-material 3D printed objects.

IMPLEMENTING TEXTURES BASED ON CELLS

Our materials consist of cells on a regular grid. In the following, we describe the design of the cells. In order to create different textures, we describe the cell parameters that can be varied by designers and their effects. For simplicity, we first describe the mechanics of the fold cell at the example of creating a simple bumpy structure. Then, we focus on the top of the cell and how to create more complex texture geometries.

Our prototypes were printed using NinjaFlex, a rubber-like filament, on an Ultimaker 2+, which is a consumer-level fused deposition modeling (FDM) 3D printer. The fold cells have a width of 15 mm and a height and depth of 7.5 mm. We provide all 3D objects source files¹ (.stl format) to allow researchers to build upon our work.

Geometry of the fold cell

Figure 8 illustrates the structure of our unitary cell, the *fold cell*. It consists of two walls that are connected to four members by living hinges, i.e., thin parts that, due to their reduced stiffness, can flex.

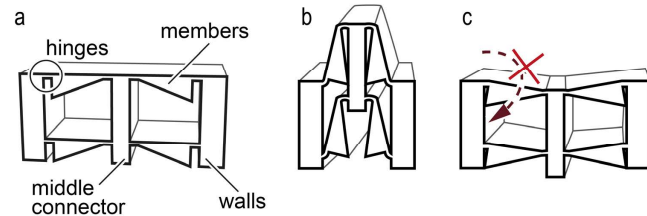


Figure 8: The fold cell consists of parametrizable walls, hinges and members that enable a stable fold to transform the material from straight to textured (here corrugated).

Compressing the cell causes the internal four-bars to shear in a pre-defined direction, here it shears upwards. The shearing direction is encoded into the triangular shape of

the members (Figure 8c): having the thicker part of the members towards the walls prevents them from tilting down as they would collide with the walls; reversing the triangular members would result in a downwards fold.

Amplitude and frequency of a texture

The height of the protruding texture is defined by the length of the fold cell’s members. As shown in Figure 9a, longer members allow the resulting texture to pop out more from the 3D object. We call this the *amplitude* of the texture.

We found the relationship of maximum amplitude and member length to be described as:

$$amplitude_{max} = \sqrt{\left(\frac{c}{2} - w - m\right)^2 - t^2}$$

where c denotes the cell width, w the wall thickness, m the middle connector thickness, and t the member thickness.

While an increased wall thickness decreases the amplitude, it simultaneously decreases the *frequency* of the texture. Figure 9b illustrates how increasing the wall thickness of the cells separates neighboring cells further apart and thus reduces the resulting texture’s frequency. To achieve a high frequency with low amplitude, we can split the cell in two, as shown in Figure 9c.

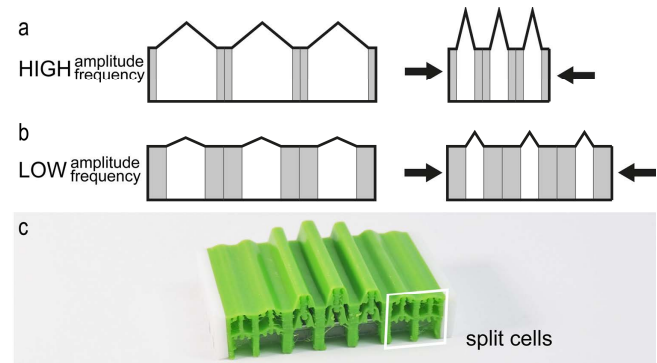


Figure 9: (a-b) The length of the fold cell’s members defines the texture’s amplitude, i.e., the height of each bump. (c) The amplitude can be varied across the material and cells can be split to create a higher frequency.

The same parameters that influence the amplitude also define the maximum compression ratio of the texture. For example, cells with thick walls can be compressed by a smaller extend than cells with thin walls. The compression ratio per cell is therefore modeled as

$$compression\ ratio_{cell} = \frac{2w+2t+m}{c}$$

Force-dependent textures

The thickness of the hinges defines how much force is required to fold them outwards. Figure 10 illustrates that because thicker hinges require more force to be deformed than thin hinges (in fact, the required force is to the power of three [9]), they also *fold later* as they are subjected to constant compression.

¹ <https://hpi.de/baudisch/projects/metamaterial-textures.html>

To embed multiple textures in one object, as demonstrated in our door handle example shown in Figure 1, designers specify a smaller hinge thickness for the cells that will pop up first (cf. the ridges in the door handle) and a larger hinge thickness for the cells that will fold later (cf. the spiky cells). This allows designers to potentially make every row dependent on different amounts of force to create animated textures on 3D objects.

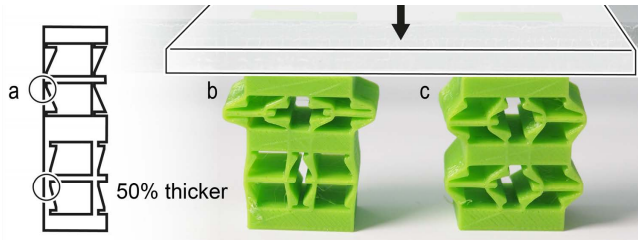


Figure 10: (a) Here, we demonstrate how our approach embeds multiple textures in one surface by varying the thickness of the hinges. (b) The upper cell’s hinges are thinner and thus fold before (i.e., under less force) than the lower cell’s hinges, which are 50% thicker. (c) If the hinge thickness is uniform across cells, they fold simultaneously under the same load.

While a more exhaustive technical evaluation is planned for future work, we report a simple experiment to evaluate an example structure, featuring a row with four different fold cells. Each cell in Figure 11 has a unique hinge thickness; from left to right 0.6 mm, 0.8 mm, 0.4 mm, 1.0 mm. Figure 11a shows the third cell folding upwards as the whole row is compressed while connected to a force gauge. As expected, we confirmed that the folding order of our printed cells is indeed dictated by their increasing hinge thickness: 0.4 mm folds at 3.0 N, 0.6 mm folds at 3.65 N, 0.8 mm folds at 6.4 N and 1.0 mm folds at 8.15 N.

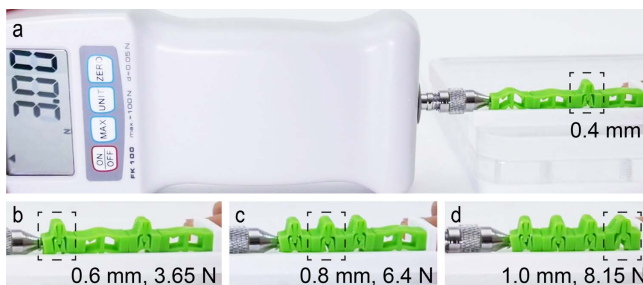


Figure 11: Our force test confirms that by varying the hinge thicknesses, we can control when the cell folds up.

Single cell primitives

We now describe how to create bumps with more expressive shapes. So far, we have seen only textures created by bumps with a triangular protrusion. Now, we demonstrate that by simply altering the hinges on the top of the single cells, we can achieve more interesting results.

Triangular, Squared and Rounded Texture bumps

The simplest bump is just a straight fold, which resembles a small triangle (Figure 12a). Next, we design a box-like texture bump by specifying equal widths for all members (Figure 12b). We can also create round bumps (Figure 12c)

by making the hinge in the middle long and double its thickness (here to 0.8 mm).

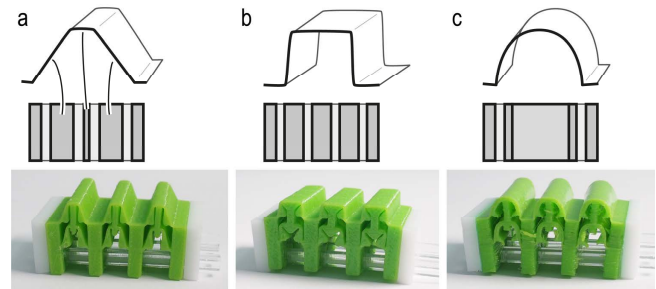


Figure 12: Variations of textures using a straight fold: (a) triangular folds, (b) box-like or (c) round bumps.

Zigzag texture bumps

The simple straight bend that we demonstrated can be transformed to create more elaborate textures. We do so by offsetting the hinge positions on both edges of the cell, as illustrated by Figure 13a. This results in a diagonal folding up. The connection to the cell’s middle connector is only possible at a small part in the middle of the hinge, which requires the connector to be tapered in the z-axis (Figure 13b). Despite the connection being very small, the structure works as before because the lower members push the cell up.

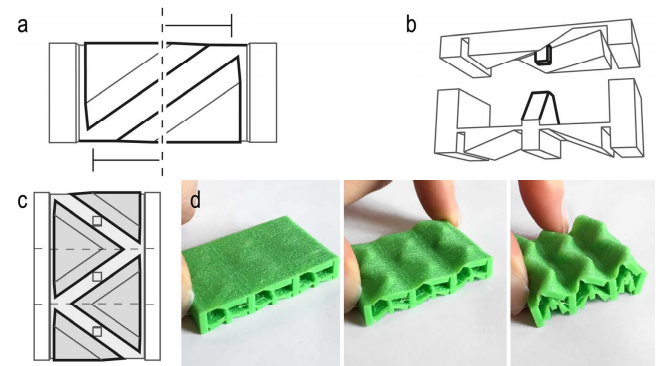


Figure 13: (a) Adding horizontal offsets to the hinge creates diagonal folds. (b) The connector width to the lower cell structure needs to be decreased gradually to connect to the cell top. (c) Composing multiple diagonal fold cells creates zigzag patterns that (d) can be varied in magnitude.

Note that the diagonal hinge also reduces the maximum amplitude and the compression ratio. This is because the offset of the hinge created members of different lengths, i.e., a shorter and a longer member. The cell can now only fold up to the extent of the shorter member.

Spiky texture bumps

To create the spiky texture from our door handle example, we take the zigzag pattern from Figure 13a and remove the material from the hinge, only leaving a thin connection in the middle as shown in Figure 14a. This results a malleable spiky texture as the triangles fold upwards but are disconnected from their walls.



Figure 14: (a) Leaving gaps between the members of the cell top (b) allows for creating spikey textures.

Diamond texture bumps

The diamond bump can be created by changing the location where the horizontal offset is effective in the vertical axis. Figure 15 shows how this allows designers to create a Y-shaped pattern, which has the effect of flattening out in the middle. By mirroring this pattern to the neighbor cell, it creates a diamond-shaped texture. It is also possible to achieve the diamond pattern on a single cell, by varying the vertical offsets on both sides (Figure 15b).

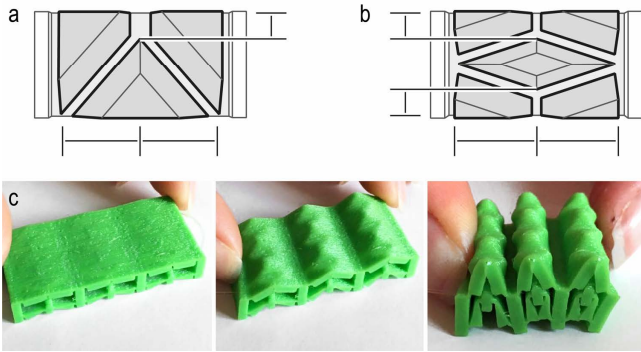


Figure 15: (a-b) Offsetting the hinge vertically and horizontally creates (c) diamond shaped textures, which flatten in the middle and thus create a different tactile feel.

INTERACTIVE EDITOR FOR TEXTURES

Figure 16 shows the interactive editor we built to assist designers in creating textures based on metamaterials. Here, we see a user creating a block that when pushed displays a zigzag texture on the top.

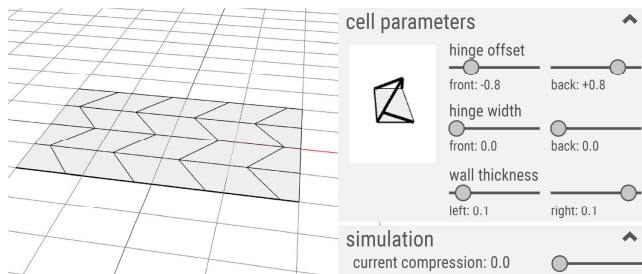


Figure 16: Our editor assists users in creating metamaterial textures. Users adjust the cell geometry using sliders and lay the cells out on the grid.

User interaction

In the interactive editor, we exploit the fact that the fold cell is fully parameterized. Users can set all parameters (Figure 16, right), for example hinge offset, width and wall thickness, simply by dragging the individual sliders. An interactive preview of a cell in its actuated state (Figure 16, left

from the parameters) allows users to see how the cell will look like once placed and actuated. After setting all parameters, users can arrange the cells on a regular grid to create textures from metamaterials.

Previewing textures by means of simulation

The editor offers a preview of the resulting texture based on the user's current metamaterial design. They can interactively preview the different textures, which result from different compression rates by simply dragging the slider (Figure 17) that specifies the current compression. The deformation of the textures is rendered in real-time.

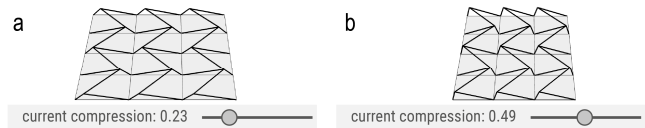


Figure 17: Users interactively preview their textures by dragging the slider that sets the simulated compression.

Software implementation

Our editor runs in a browser. It is based on the editor for metamaterial mechanisms [15], which is built using *node.js* (a Javascript runtime framework) and utilizes the *three.js* library for rendering.

The simple kinematic simulation that we implemented in our editor allows for previewing, in real-time, how the designed textures fold up when the material is compressed via a GUI slider. Our simulation calculates only geometric transformations by simple propagation, i.e., as a cell compresses, its members move, in turn, these members move the neighboring cell's members, and so forth.

Note that at this stage our simulation does not take material properties into account. We opted for this approach as it allows for an interactive simulation (at 30 fps) compared to, e.g., finite element analysis, which is computationally more expensive and therefore slower, but more accurate.

Generating a printable file

To simplify the process of creating a texture based on metamaterials, our editor only requires users to choose how the surface of the cell should fold to form their texture. In fact, the remainder of the 3D object, e.g., its internal structure beyond the top layer, is generated by our editor when users export the final texture.

We derive the parameters for the remainder cell geometry from the user defined hinge positions. The wall thickness is derived from the hinge offsets. For a diagonal hinge, we gradually decrease the thickness of the middle connector towards the top so that it connects only to the center of the texture geometry with minimum widths of 0.3 mm. After having created the cell body for the fold cell, we repeat this structure until the user-defined volume is filled. Finally, this geometry is exported into a 3D printable *.stl* file.

Limitations of the interactive editor

Our editor only allows creating objects with simple geometries, i.e., planar, curved, and cylindrical. Our current kine-

matic simulation enables real-time interaction at the expense of offering more simplistic results. These, however, preview all the textures we demonstrated correctly.

DISCUSSION, LIMITATIONS AND OUTLOOK

In the following we present a discussion of our prototype centered on its limitations and potential implications.

Limitations

While we see this work as the first step towards creating textured objects using metamaterials, it certainly has a number of limitations. The most evident one is that our approach works only for objects in which exact dimensions are not critical. The exact length of a door handle, for example, is not critical for its functionality. In fact, our approach always generates a change in the object's overall shape. Secondly, our current approach is limited to actuation using a global force pulled along one dimension (e.g., when the user pulls the wires to configure the shoe sole). In the future, we want to investigate dynamic textures that can be actuated in two and three dimensions. Furthermore, we currently use external materials, such as the strings that are pulled, to actuate the metamaterial textures. In the future, we want to integrate the actuation with the metamaterial itself so that it can be fabricated in one piece. Lastly, our demonstration objects are limited in that the textures pop out on planes or cylindrical shapes only. Ideally, textures would be integrated in arbitrary shapes.

Alternative forms of actuation

Since in this paper, we explore the idea of materials that exhibit textures by means of metamaterials, we opted to actuate our textures in the simplest way (e.g., pushing, strings, etc.), so that no electronic components are required (such as motors, batteries, microcontrollers, etc.). However, alternative actuation mechanisms can certainly be used, e.g., motors, that will allow programmatic real-time behavior without users' actions. For instance, one can envision how the door handle changes texture automatically by being digitally connected to the user's calendar.

Outlook for nano-scale metamaterial textures

The scale of our current textures was dictated by the resolution of our consumer-grade desktop printer. However, using state of the art high-resolution printers (e.g., nanoscribe²) it might be possible to uncover new opportunities using our approach. Firstly, by scaling down a design similar to that of Figure 9a, i.e., with long members that protrude a lot outside the object, it might be possible to use our approach to generate furry textures [19, 31] that are transformable.

Conversely, using high-resolution printers to fabricate cells with short members would yield a texture made from micro bumps that would feel "rough" to the user. This could also be employed as a prototyping tool to alter an object's friction when sliding on a surface.

CONCLUSIONS

We proposed an approach that leverages metamaterials to create transformable textures on 3D printed objects. We demonstrated the benefits of our approach in three objects and provided an interactive editor to allow researchers and users to create novel textures.

We see metamaterial textures as a first step to integrate transforming textures into 3D printed objects. In the long run, we think such an approach might be relevant to disseminate more expressive haptics in everyday objects. We hope this opens new dialogs between UX and product designers and results in novel everyday objects with multiple pre-integrated textures that can be activated by the end user.

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